# DICK SCHAFF MATH SUPERBOWL XLVII
# LEVEL 5 BLITZ: SECONDARY MATH III-2022

1) The sum of the roots of the equation $3x^2 - 2x + 1 = 0$ is:

a) $\frac{2}{3}$ b) $\frac{-1}{3}$ c) $\frac{-2i}{3}$ d) $\frac{i}{3}$ e) none of these

2) $\sqrt[6]{81x^2}$, $x \geq 0$ is equivalent to

a) $\sqrt{9x}$ b) $\sqrt[3]{9x}$ c) $\sqrt{9x^3}$ d) $\sqrt[3]{27x^{\frac{1}{3}}}$ e) none of these

3) When factoring the expression, $8x^3 + y^6$ using the "sum of two cubes" method, one of the terms of one of the factors is

a) $2xy$ b) $4xy^2$ c) $4x^2y$ d) $y^3$ e) none of these

4) The expression $\frac{x+y}{\frac{1}{x}+\frac{1}{y}}$ is equivalent to

a) $xy$ b) $\frac{1}{xy}$ c) $x^2y^2$ d) $\frac{xy}{x+y}$ e) none of these

5) The remainder obtained by the division $(x^4 - x^3 + 2x^2 + 1) \div (x - 2)$ is

a) 9 b) -9 c) 17 d) -17 e) none of these

6) The real part of the product $(3 - i)(2 + 3i)$ is

a) 3 b) -3 c) 9 d) -9 e) none of these

7) The number of horizontal asymptotes of the graph of $f(x) = \frac{|x|}{x+1}$ is

a) 0 b) 1 c) 2 d) 3 e) none of these

8) If $\sin(t) = \frac{-3}{8}$ and $\pi < t < \frac{3\pi}{2}$, then $\tan(t) =$

a) $\frac{\sqrt{55}}{3}$ b) $\frac{3}{\sqrt{55}}$ c) $\frac{-\sqrt{55}}{8}$ d) $\frac{-3}{\sqrt{55}}$ e) none of these

9) The weight of water that must be evaporated from 40 pounds of a 20% salt solution to produce a 50% salt solution is

a) 24 pounds b) 18 pounds c) 12 pounds d) 6 pounds e) none of these

10) The value of $x$ for which $f(x) = -2x^2 - 12x + 1$ is a maximum is

a) -6 b) 6 c) -12 d) 12 e) none of these

11) The product of the solutions to the equation $x^4 - 3x^2 - 4 = 0$ is

a) -4 b) 4 c) $-4i$ d) $4i$ e) none of these

12) The antilogarithm of $x$ is the number $y$ such that $\log y = x$. According to this definition, the antilogarithm of 3 is

a) 8 b) $3^{10}$ c) $10^3$ d) $\log 3^{10}$ e) none of these

13) The value of $e$ to one decimal place is

a) 3.1 b) 2.7 c) 2.3 d) 1.4 e) none of these

14) In lowest terms, the denominator of the solution to the equation $2(4^{x+1}) = 8^{3x}$ is

a) 1 b) 2 c) 6 d) 7 e) none of these

15) In lowest terms, the numerator of the rational form of the recurring decimal $0.2\overline{3}$ is

a) 7 b) 21 c) 23 d) 27 e) none of these

16) If a fraction $\frac{x}{y}$ is reduced to lowest terms then the result is $\frac{4}{5}$. When the numerator of $\frac{x}{y}$ is increased by 4 and the denominator is increased by 10 then the new fraction is $\frac{2}{3}$. The value of $x$ is

a) 4 b) 5 c) 12 d) 16 e) none of these

17) If $\theta = \text{Arctan}\left(\frac{-12}{5}\right)$ then $\sin\left(\frac{\theta}{2}\right) =$

a) $\frac{-12}{13}$ b) $\frac{-3}{\sqrt{13}}$ c) $\frac{2}{\sqrt{13}}$ d) $\frac{-2}{\sqrt{13}}$ e) none of these

18) If $f(x) = x + 1$, then $\frac{1}{f(x)} \cdot f\left(\frac{1}{x}\right)$ is equivalent to

a) 1 b) $\frac{1}{x}$ c) $x^2$ d) $\frac{x}{x+1}$ e) none of these

19) The $y$ coordinate of the vertex of the parabola described by the function $f(x) = 2x^2 - 4x + 1$ is

a) -2 b) 2 c) -1 d) 1 e) none of these

20) A line contains a point at (-5, -6) and has the $x$-intercept at 4. The slope of the line is

a) $\frac{2}{3}$ b) $\frac{3}{4}$ c) $\frac{5}{6}$ d) $\frac{6}{5}$ e) none of these

21) $\sin^2(-5x) =$

a) $-\sin^2(-5x)$ b) $\frac{1+\cos(10x)}{2}$ c) $\frac{1-\sin(10x)}{2}$ d) $\frac{1-\cos(10x)}{2}$ e) none of these

22) $2^{-1} + 2^{-2}$ is equal to

a) $2^{-3}$ b) 4 c) $\frac{1}{8}$ d) 8 e) none of these

23) $4^x + 4^x + 4^x + 4^x$ is equal to

a) $4^{4x}$ b) $16^x$ c) $4^{x^4}$ d) $4^{x+1}$ e) none of these

24) Which of the following angles is coterminal with $\frac{-13\pi}{3}$?

a) 120° b) 600° c) $-240°$ d) $-60°$ e) none of these

25) When the non zero solution to $2x\sqrt{x} = x$ is written in lowest terms, the denominator is

a) 1 b) 2 c) 4 d) 8 e) none of these

26) If $\log_3(x) = y$, then $\log_9(x) =$

a) $2y$ b) $-y$ c) $\frac{y}{2}$ d) $\sqrt{y}$ e) none of these

27) The lines $l1$ and $l2$ intersect at (5, -2) and lines $l2$ and $l3$ intersect at (-3, 3). The slope of line $l2$ is

a) $-\frac{5}{2}$ b) $-\frac{5}{8}$ c) $-\frac{2}{5}$ d) $\frac{1}{2}$ e) none of these

28) How many solutions does the equation $\cos^2(3x) = 0.38$ have for $0 \leq x < 2\pi$

a) 6 b) 2 c) 12 d) 4 e) none of these

29) Consider the equation $(x-2)^2 + (y+1)^2 = 16$. For how many values of $x$ does $y = 4$?

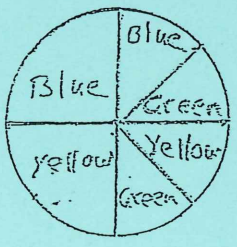
a) 0 b) 1 c) 2 d) 4 e) none of these

30) How does the graph of $y = f(x+2)$ compare to the graph of $y = f(x)$?

a) Horizontal shift of 2 units right b) Vertical shift of 2 units up c) Vertical stretch by a factor of 2
d) Horizontal stretch by a factor of 2 e) none of these

31) A dartboard (see diagram) is a circle where the corresponding central angles are 90 degrees or 45 degrees. If a dart is thrown at random at the dart board, then what is the probability that a blue region is hit?

a) $\frac{1}{4}$ b) $\frac{1}{3}$ c) $\frac{1}{2}$ d) $\frac{2}{5}$ e) none of these



32) The exact value of $\csc(75°) =$

a) $\sqrt{6}-\sqrt{2}$ b) $\frac{4}{\sqrt{6}-\sqrt{2}}$ c) $\sqrt{2}+\sqrt{6}$ d) $\frac{\sqrt{6}-\sqrt{2}}{4}$ e) none of these

33) Function $f$ is a linear function such that $f(2) = 3$ and $f(4) = 1$. The value of $f(-101)$ is

a) -72 b) -58 c) 92 d) 106 e) none of these

34) The diagonal measurement of a square is $6\sqrt{2}$. The perimeter of the square is

a) 24 b) $12\sqrt{2}$ c) 12 d) 20 e) none of these

35) The relationship between the number of decibels $\beta$ and the intensity of a sound $I$ in watts per square meter is $\beta = 10 \log_{10}\left(\frac{I}{10^{-12}}\right)$ The number of decibels of a sound with an intensity of $10^{-2}$ watts per square meter is

a) $10^{11}$ b) $10^{-9}$ c) 1 d) 100 e) none of these

36) The focus of the parabola described by the equation $x = -8y^2 + 48y - 71$ is:

a) (1, 3) b) (3, 3) c) (-1, 3) d) (1, 1) e) none of these

37) log 3 is equivalent to

a) (ln 3)(ln 10) b) $\frac{\ln 3}{\ln 10}$ c) $\frac{\ln 10}{\ln 3}$ d) 3 ln 10 e) none of these

38) The inverse of $f(x) = \frac{x-2}{2x+1}$ is

a) $\frac{x-2}{2x-1}$ b) $\frac{x+2}{1-2x}$ c) $\frac{x-2}{1-2x}$ d) $\frac{x+2}{2x+1}$ e) none of these

39) The product of the $y$ values of all the solutions to the system $x^2 + y^2 = 13$, $x^2 - y^2 = 5$ is

a) 1 b) 16 c) 27 d) 64 e) none of these

40) How many ways can three envelopes be placed in five mailboxes if no mailbox may contain more than one envelope?

a) 27 b) 125 c) 6 d) 3 e) none of these

41) When expanding the binomial $(2x - 1)^7$ the numerical coefficient of $x^5$ is

a) 21 b) 32 c) 672 d) 1344 e) none of these

42) The value of $\sum_{i=2}^{\infty}(\frac{2}{5})^i$ is

a) $\frac{2}{3}$ b) $\frac{3}{2}$ c) $\frac{2}{5}$ d) $\frac{4}{15}$ e) none of these

43) If it takes 10 minutes to fill $\frac{5}{12}$ of a hole, then the number of minutes to fill the rest of the hole at this rate is

a) 14 b) 24 c) 30 d) 48 e) none of these

44) The expression $\frac{x-4\sqrt{x}+4}{\sqrt{x}-2}$ is equivalent to

a) $\sqrt{x}$ b) $2\sqrt{x}$ c) $\sqrt{x}+2$ d) $\sqrt{x}-2$ e) none of these

45) Which of the following is the equation of an asymptote of the graph of the function $y = \sec(2x)$

a) $x = \frac{\pi}{2}$ b) $x = \frac{-\pi}{4}$ c) $x = \frac{-\pi}{2}$ d) $x = \frac{3\pi}{2}$ e) none of these